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On the Study of $ZM(G)$ the Central Measure Algebra of a Connected Lie Group

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ABSTRACT. In this paper we reduce the study of connected Lie groups to the study of connected Lie groups with no compact normal semisimple subgroups. We also reduce the study of $ZM(G)$ to the study of $Z^G M(B(G))$, the algebra of G-invariant bounded measures on the given characteristics subgroup $B(G)$ of G. Finally, $B(G)$

$$(1) \to T^k \to B(G) \to R^m \times D \to (1)$$

Where T is the unit circle in the complex plane and D is a discrete finitely generated abelian group.

1. Introduction and Notations

We denote the Banach space of complex, finite, regular, Borel measures on a locally compact Hausdorff space X by $M(X)$.

If S is a locally compact topological semigroup, then $M(S)$ is a Banach algebra under the operation $(\mu, \nu) \to \mu * \nu$ (convolution) defined by the condition

(1.1) $$\int f \, d\, \mu * \nu = \int\int f(st)\, d\, \mu(s)\, d\, \nu(t)\ , f \in C_o(S)$$

Recall that $M(S)$ is the dual space of $C_o(S)$, the space of continuous functions vanishing at infinity on S; hence (1.1) defines a measure $\theta * \nu$ in $M(S)$.

It is well known that convolution is associative and distributive and satisfies $\| \mu * \nu \| = \| \mu \| \| \nu \|$. Hence $M(S)$ is a Banach algebra under convolution. Moreover, $M(S)$ is commutative if and only if S is abelian. Furthermore, if S has an identity e, then the point mass δ_e at e is an identity for $M(S)$.

It follows from the standard facts of integration theory that (1.1) holds for all bounded Borel functions f if and only if it holds for functions in $C_o(S)$. In particular, with $f = \chi_E$, the characteristics function E, we have

(1.2) $$\mu * \nu(E) = \int \int \chi_E(st)\, d\,\mu(s)\, d\nu(t)$$

If G is a locally compact group, then the measure algebra $M(G)$ has an identity δ_e and the inversion map $g \rightarrow g^{-1}$ induces an involution $\mu \rightarrow \widetilde{\mu}$ *on* $M(G)$ *defined by* $\widetilde{\mu}(E) = \mu(E^{-1})$.

The center $ZM(G)$ of $M(G)$ consists of all $\mu \epsilon M(G)$ such that $\mu * \nu = \nu * \mu$ for all $\nu \epsilon M(G)$. Then $ZM(G)$ is a commutative Banach algebra under convolution even though the underlying group G need not be commutative.

2. The Operator $U^{\#}$

Let G be a locally compact group. For a compact normal subgroup K or G, let w_K be the normalized Haar measure over K. Hence we shall write

(2.1) $$w_K(f) = \int_K f(t)\, dt \quad , \quad f \epsilon C_o(G)$$

Lemma 2.1

Let G be a locally compact group, then for any compact subgroup K of G, we have, $w_K \epsilon ZM(G)$.

Proof

Since w_K is the Haar measure on K, then w_K is a Lebesque measure on G, as the trivial extension of w_K on G, and

2) $$\int f(xtx^{-1})\, dt = \Delta(x) \int f(t)\, dt$$

ere $\Delta : G \rightarrow R$ is the continuous homomorphism modular function corresponding

(2.2) $$\int f(xtx^{-1})\, dt = \Delta(x) \int f(t)\, dt$$

where $\Delta : G \rightarrow R$ is the continuous homomorphism modular function corresponding to w_K on G.

Let $f = \chi_K$ be the characteristic function of K. We have $xKx^{-1} = K$, for each x in G, since K is a normal subgroup. But then $X_K(xtx^{-1}) = \chi_K(t)$. This implies

(2.3) $$\int_K \chi_K(xtx^{-1})\, dt = \int_K \chi_K(t)\, dt \, , \qquad \text{for each } x \text{ in } G.$$

Hence $\Delta(x) = 1$, for each x in G, and

(2.4) $$\int_K f(xtx^{-1})\, dt = \int_K f(t)\, dt \, , \qquad \text{for each } f \text{ in } C_o(G).$$

i.e. $\delta_x * w_K * \delta_x{-1}\,(f) = w_K(f)$, for each f in $C_o(G)$ and x in G, where δ_x is the point mass at x. Hence $\delta_x * w_K * \delta x^{-1} = w_K$. So by Greenleaf, *et al.*[1], $w_K \epsilon ZM(G)$.

Corollary 2.2

For G, K and t as above, we have

(2.5) $\int_K f(tx)\, dt = \int_K f(xt)\, dt$, for each x in G and f in $C_o(G)$.

Now let $L_x : f \to f_x -1$ (respect. $R_x : f \to f^x$) be the left (respect. right) representation of G, where $f_x(y) = f(xy)$ and $f^x(y) = f(yx)$. Then L_x and R_x can be regarded as strong continuous representations of G on $L^l(G)$. So (2.5) can be rewritten as

(2.6) $\int_K f^x(t)\, dt = \int_K f_x(t)\, dt$, *for each* f in $C_o(G)$ and x in G.

Also since the Haar measure, w_K of the compact normal subgroup K of G, is invariant under translation, we get

(2.7) $\int_K f^x(t)\, dt = \int_K f^y(t)\, dt$, for each f in $C_o(G)$, x in G and y *in* $\bar{x}$.

where $\bar{x} = Kx = \{ tx : t \in K \}$.

Now if f is a continuous function on $G$and $x \in G$, we shall write

(2.8) $f^{\#}(\bar{x}) = \int_K f^x(t)\, dt$

Denote by $U^{\#}$ the linear mapping $f \to f^{\#}$. The following is a slight extension of a result due to Halmos[2] (see Theorem D p. 279).

Proposition 2.3

Let G, K and $U^{\#}$ be as above, then

(1) $f^{\#}$ is continuous. If f is left (resp. right) uniformly continuous, so is $f^{\#}$.
(2) If f is bounded, so is $f^{\#}$ and $\| f^{\#} \|_{G/K} \leq \| f \|_G$.
(3) If f is positive definite, so is $f^{\#}$.
(4) If $f \in C_o(G)$, then $f^{\#} \in C_o(G/K)$ and the linear mapping

$$U^{\#} : C_v(G) \to C_o(G/K)$$

is onto.

Proof

It is easy to see that the translation map $KxG \to G$ is continuous, since G is a topological group. If F is any compact set, $KF = FK$ is also compact, since K is compact and normal. Hence

$$\begin{aligned} \| f^{\#} \|_{KF} &= \sup \{ | f^{\#}(\bar{x}) | : \bar{x} \in KF \}. \\ &= \sup \{ | \textstyle\int_K f(tx)\, dt | : \bar{x} \in KF \}. \\ &\leq \sup \{ \textstyle\int_K | f(tx) |\, dt : \bar{x} \in KF \}. \\ &\leq \sup \{ | f(tx) | : t \in K \text{ and } x \in F \\ &= \| f \|_{KF} < \infty \end{aligned}$$

Applying the above relation, to $F = \{ x \}$, shows that $f^{\#}$ is well-defined and that f is bounded. Therefore, $\| f^{\#} \|_{G/K} \leq \| f \|_G$.

It is clear that if the support of f is compact, then the support of $f^{\#}$ is contained in the compact set KF. For the continuity, assume first that f is left uniformly continuous and let $\varepsilon > 0$. Then there is a compact neighbourhood V of e in G such that $y^{-1}x \in V$ implies $|f(y) - f(x)| < \varepsilon$. Now for each $z \in G$ we have $(zy)^{-1}zx = y^{-1}x \in V$, *i.e.* $|f(y) - f(x)| < \varepsilon$ implies $|f(zy) - f(zx)| < \varepsilon$, for each z in G. So $y^{-1}x \in V$ implies

$$\begin{aligned} |f^{\#}(\bar{y}) - f^{\#}(\bar{x})| &= |\textstyle\int_K (f(ty) - f(tx))\, dt| \\ &\leq \textstyle\int_K |f(ty) - f(tx)|\, dt \\ &\leq \textstyle\int_K \varepsilon dt = \varepsilon \end{aligned}$$

Suppose now f is merely continuous and let $x \in G$. If W is a fixed compact neighbourhood of x, then $V = KW$ is a compact neighbourhood of x such that $tx \in V$, for each $t \in K$. By Urysohn's lemma, there is a (uniformly) continuous function g with compact support, which is equal to f on V, hence

$$g^{\#}(\bar{x}) = \int_K g(tx)\, dt = \int_K f(tx)\, dt = f^{\#}(\bar{x})$$

Since $g^{\#}$ is uniformly continuous, $f^{\#}$ is continuous at $\bar{x}$.

It is obvious from the definition that if f is positive definite, then so is $f^{\#}$. It is easy now to see that whenever f vanishes at infinity then so does $f^{\#}$, *i.e.* $f \in C_o(G)$ implies $f^{\#} \in C_o(G/K)$.

Let Q be a Banach algebra, I be an ideal of Q and $Z(Q)$ be the subalgebra of all central elements of Q. If ΔQ (resp. ΔI) is the spectrum of Q (resp. I) *i.e.* the set of all non-zero homomorphisms of Q (resp. I) onto C, then we have

Lemma 2.4

Let Q be a Banach algebra and I be an ideal of Q. Then

$$\Delta I = \{ h \in \Delta Q : h(I) \neq 0 \}$$

Proof

Let $h \in \Delta I$, then there is a $j \in I$ such that $h(j) = 1$ (otherwise $h(I) = 0$). So, for every a in Q, define $h_Q(a) = h(aj)$. Then it is easy to check that h_Q is a homomorphism of Q, and where $h_{Q/I} = h$. So the restriction map : $\Delta Q \to \Delta I$ is onto and the lemma follows easily.

Suppose now that $U : M(G/K) \to M(G)$ is the adjoint mapping of $U^{\#} : C_o(G) \to C_o(G/K)$, *i.e.* for any measure $\lambda \in M(G/K)$ and any continuous function $f \in C_o(G)$, we have $U(\lambda)(f) = \lambda(f^{\#})$. In other words, if $\mu = U(\lambda) \in M(G)$ and $f \in C_o(G)$, we get

(2.9) $$\int_G f(x)\, d\mu(x) = \int_{G/K} \int_K f(tx)\, dt\, d\lambda(\bar{x})$$

We conclude this section by the following theorem which we need later.

Theorem 2.5

Let G be a locally compact group and K be a compact normal subgroup of G. Then there exists a Banach algebra homomorphism which maps $M(G/K)$ into $M(G)$ and maps δ_K, the identity of $M(G/K)$, to $w_K \in M(G)$, the Haar measure of K. Moreover, we have

(1) $M(G/K) = M(G) * w_K$, hence $M(G/K)$ is an ideal of $M(G)$.
(2) $ZM(G/K) = ZM(G) * w_K$, hence $ZM(G/K)$ is an ideal of $ZM(G)$.
(3) $\Delta M(G/K) = \{ h \in \Delta M(G) : h(w_K) = 1 \}$.
(4) $\Delta ZM(G/K) = \{ h \in \Delta ZM(G) : h(w_K) = 1 \}$.

Proof

Suppose that $U : M(G/K) \to M(G)$ is the adjoint mapping of $U^{\#} : C_o(G) \to C_o(G/K)$, see prop. 2.3 and equation (2.9) above. Since $U^{\#}$ is a linear mapping which is onto this implies that U is a linear mapping and that U is one - to - one.

To show that U preserves the convolution, one needs to observe that whenever $\mu = U(\lambda)$ for some $\lambda \in M(G/K)$, then $\mu(f) = \lambda(f^{\#})$ for all f in $C_o(G)$, *i.e.* (2.9) is given. Now for $\lambda_1, \lambda_2 \in M(G/K)$, let $\mu_1, \mu_2 \in M(G)$ such that $\mu_1 = U(\lambda_1)$ and $\mu_2 = U(\lambda_2)$. Also let $\mu = U(\lambda_1 * \lambda_2)$. So we need to prove that $\mu = \mu_1 * \mu_2$, *i.e.* $U(\lambda_1 * \lambda_2) = U(\lambda_1) * U(\lambda_2)$. For this purpose, let $f \in C_o(G)$, then

$$\begin{aligned} \mu_1 * \mu_2(f) &= \int_G \int_G f(xy)\, d\mu_1(x)\, d\mu_2(y) \\ &= \int_G \{ \int_G f^y(x)\, d\mu_1(x) \}\, d\mu_2(y) \end{aligned}$$

Let $g(y) = \int_G f^y(x)\, d\mu_1(x)$, thus, by definition of μ_1, one gets

$$\begin{aligned} g(y) &= \int_{G/K} \int_K f^y(tx)\, dt\, d\lambda_1(\bar{x}) \\ &= \int_{G/K} \int_K f(txy)\, dt\, d\lambda_1(\bar{x}) \end{aligned}$$

It is easy to see that $g(sy) = g(y)$, for each s in K, since the Haar measure is translation invariant modulo K.

Now

$$\begin{aligned} \mu_1 * \mu_2(f) &= \int_G \{ \int_G f^y(x)\, d\mu_1(x) \}\, d\mu_2(y) \\ &= \int_G g(y)\, d\mu_2(y) \\ &= \int_{G/K} \int_K g(sy)\, ds\, d\lambda_2(\bar{y}) \quad , \quad (\text{since } \mu_2 = U(\lambda_2)) \\ &= \int_{G/K} g(y)\, d\lambda_2(\bar{y}) \end{aligned}$$

Since the Haar measure is normalized and g is K-invariant. Hence

$$\mu_1 * \mu_2(f) = \int_{G/K} \int_{G/K} f^{\#}(\overline{xy})\, d\lambda_1(\bar{x})\, d\lambda_2(\bar{y})$$

And since $\overline{xy} = \overline{xy}$, we get

$$\begin{aligned} \mu_1 * \mu_2(f) &= \int_{G/K} \int_{G/K} f^{\#}(\overline{xy})\, d\lambda_1(\bar{x})\, d\lambda_2(\bar{y}) \\ &= (\lambda_1 * \lambda_2)\, (f^{\#}) \\ &= U(\lambda_1 * \lambda_2)\, (f) \end{aligned}$$

But f is an arbitrary continuous function in $C_o(G)$, so we have

$$U(\lambda_1) * U(\lambda_2) = \mu_1 * \mu_2 = U(\lambda_1 * \lambda_2)$$

Hence U is a Banach algebra homomorphism.

(1) It is easy to see that for any measure ν, $\nu \epsilon M(G) * w_K = \{\mu * w_K : \mu \epsilon M(G)\}$ if and only if $\nu = \nu * w_K$ (since w_K is idempotent).

Now let $\lambda \epsilon M(G/K)$ and $\mu = U(\lambda) \epsilon M(G)$. Then for any $f \epsilon C_o(G)$, we have

$$\begin{aligned} \mu * w_K(f) &= w_K * \mu(f) \\ &= \textstyle\int_G \int_K f(tx)\, dt\, d\mu(x) \\ &= \textstyle\int_G f(\overline{x})\, d\mu(x) \end{aligned}$$

Let $g(x) = f^{\#}(\overline{x})$ for each x in $\overline{x}$, then g is constant on each coset $\overline{x} = Kx$. Also g is an element of $C_o(G)$, since f is. Moreover $g^{\#}(\overline{x}) = f^{\#}(\overline{x}) = g(x)$, for each x in $\overline{x}$, for each $\overline{x}$ in G/K. Now we can write

$$\begin{aligned} \mu * w_k(f) &= \textstyle\int_G f^{\#}(\overline{x})\, d\mu(x) \\ &= \textstyle\int_G g(x)\, d\mu(x) \\ &= \textstyle\int_{G/K} g^{\#}(\overline{x})\, d\lambda(\overline{x}) && (\text{see } \mu = U(\lambda)) \\ & && [\text{see } (2.8) \text{ and } (2.9)] \\ &= \textstyle\int_{G/K} f^{\#}(\overline{x})\, d\lambda(\overline{x}) \\ &= \textstyle\int_G f(x)\, d\mu(x) && [\text{by } (2.8) \text{ and } (2.8)] \\ &= \mu(f) \end{aligned}$$

But this means that $\mu * w_K = \mu$, *i.e.* $U(\lambda) * w_K = U(\lambda)$ for any $\lambda \epsilon M(G/K)$. So $U(M(G/K)) = M(G) * w_K$ and hence $M(G/K)$ can be regarded as an ideal of $M(G)$.

(2) Let $\lambda \epsilon ZM(G/K)$, x in G and f in $C_o(G)$ be arbitrary elements.

Then

$$\begin{aligned} \delta_x * U(\lambda) * \delta x^{-1}\ (f) &= \textstyle\int_G f(xyx^{-1})\, dU(\lambda)\,(y) \\ &= \textstyle\int_{G/K} \int_K f(xtyx^{-1})\, dt\, d\lambda(\overline{y}) \end{aligned}$$

Since the Haar measure w_K is central, we get

$$\begin{aligned} \delta_x * U(\lambda) * \delta x^{-1}\ (f) &= \textstyle\int_{G/K} \int_K f(txyx^{-1})\, dt\, d\lambda(\overline{y}) \\ &= \textstyle\int_{G/K} f^{\#}(\overline{xyx^{-1}})\, d\lambda(\overline{y}) \\ &= (\delta \ * \lambda * \delta\overline{x}^{-1}\ (f^{\#}) \end{aligned}$$

As $\lambda \epsilon ZM(G/K)$, thus $\delta x^{-} * \lambda * \delta x^{-1} = \lambda$, and $\delta_x * U(\lambda) * \delta x^{-1}(f) = \lambda(f^{\#}) = U(\lambda)(f)$. Therefore $U(\lambda)\ \epsilon ZM(G)$. Hence we have $ZM(G/K) \cong ZM(G) * w_K$.

To prove (3) and (4), let $I = M(G/K) \cong M(G) * w_K$ and $J = ZM(G/K) \cong ZM(G) * w_K$. Then I is an ideal of $M(G)$ and J is an ideal of $ZM(G)$. Applying lemma 2.5, we get $h \epsilon \Delta I$ (resp. $h \epsilon \Delta J$) if and only $h(I) \neq 0$ (resp. $h(J) \neq 0$). Hence (in both cases) $h(w_K) \neq 0$. But w_K is idempotent, so $h(w_K) = 1$, as required.

3. The Reduction of G

In this section we use Ragozin's work[3], to reduce the general case, where G is a connected Lie group, to the case where a connected Lie group has no normal compact semisimple connected subgroup.

The next theorem is proved by Iwasawa[4], theorem 2, p. 515

Theorem 3.1

Let G be a connected topological group and K a compact normal subgroup of G. If we denote by $Z(G,K)$ the centralizer of K in G, we have

$$G = K.Z(G,K)$$

Lemma 3.2

If G is a connected Lie group, then G contains a maximal compact normal semisimple (so connected) subgroup (which may be (1)).

Proof

Let Q be a semilattice of compact normal semisimple (connected) subgroups of G, then Q has a zero as follows: let n be the maximum dimension of elements of Q and K' be an element of Q with dimension n. Then if K is in Q, KK' is in Q and KK' contains K'. But $\dim(KK') \leq \dim(K')$. So $KK' = K'$.

Thus K' is a zero of Q and is the required maximal subgroup of G.

Theorem 3.3

If G is a connected Lie group, and K is the maximal compact normal semisimple subgroup of G, then

$$G = (KxZ(G,K)_o)/F$$

where $Z(G,K)_o$ is the connected component of $Z(G,K)$, the centralizer of K in G, and F is a finite subgroup of $Z(K)$, the center of K.

Proof

We have, see theorem 3.1 above, that $G = K.Z(G,K)$, since K is a compact normal subgroup. Now let $p : G \to G/K$ be the projection map. Then we have $p : Z(G,K) \to G/K$ is onto. By the open mapping theorem, it follows that p is an open map. Since $Z(G,K)_o$ is open in $Z(G,K)$, therefore $p(Z(G,K)_o)$ is open in G/K. Hence it is clear now that $G = K.Z(G,K)_o$. We also have $K \cap Z(G,K) = Z(K)$. So $K \cap Z(G,K)_o = F$ is a finite subgroup of $Z(K)$.

Now we can write G as the direct product $KxZ(G,K)_o$ modulo F, *i.e.* $G = (KxZ(G,K)_o)/F$.

Now let w_F be the Haar measure on F. In this case we have :

Theorem 3.4

For G, K and F as above

(1) $ZM(G) = ZM(KxZ(G,H)_o) * w_F$.
(2) $\Delta ZM(G) = \{ h \epsilon \Delta ZM(KxZ(G,K)_o) : h(w_F) = 1 \}$.

Proof

Since F is a finite central subgroup of K, the maximal compact normal semisimple subgroup of G, therefore F is a compact normal subgroup of $KxZ(G,K)_o$. Applying (2) and (4) of theorem 2.6, the proof follows easily.

By 3.4, if we know $\Delta ZM(KxZ(G,K)_o)$, we know $\Delta ZM(G)$. But results of Ragozin[5] reduce the study of $\Delta ZM(KxZ(G,K)_o)$ to the sudy of $\Delta ZM(Z(G,K)_o)$. In fact concerning the spectrum (the maximal ideal space) of $ZM(SxH)$, the center of the measure algebra of SxH where S is a compact simple Lie group and H an arbitrary locally compact group, it has been proved that :

$$\Delta ZM(SxH) = \Delta ZM(S)\, x\Delta ZM(H)$$

(This together with earlier results of Ragozin[3], yield a complete description of the spectrum $\Delta ZM(K)$ for any compact connected semisimple Lie group (K).

Ragozin[3], shows that if S is a compact simple Lie group, then :

$$\Delta M(S) = \hat{S} U \hat{Z}$$

where Z is the center of S and $\hat{S}$ (resp. $\hat{Z}$) is the dual space of S (resp. Z).

So we need to study $ZM(G)$, where $G(=Z(G,K)_o)$ is a connected Lie group with no compact normal semisimple subgroups.

Standing Assumption 3.5

In the remainder of this paper, G will be a connected Lie group with no compact normal semisimple subgroups.

4. The Structure of $B(G)$

In this section as a conclusion, we reduce the study of $ZM(G)$, by Greenleaf[1], to the study of $Z^G M(B(G))$, the algebra of G-invariant bounded measures on $B(G)$, the subgroup of G of elements with relatively compact conjugacy classes. We also define the structure of $B(G)$.

Tits in[6], theorem (1) and corollary (1), shows that if G is a connected locally compact group, then $B(G)$, the subgroup of G of elements with relatively compact conjugacy classes, is a closed characteristic subgroup in G.

If we consider G/K, where K is the maximal connected compact normal subgroup, then, according to theorem (1) of Tits[6],

$$B(G/K) = B(G/K)_o \cdot Z(G/K)$$

Since $B(G/K)_o$ is a vector group and $Z(G/K)$ is compactly generated, therefore $B(G/K)$ is a compactly generated abelian group. Now $B(G/K)$ has no compact connected subgroup. Hence $B(G/K) = V \times D$, where V is a vector group (R^m for some integer m) and D is a discrete finitely generated abelian group on which the G-inner automorphisms act trivially.

Hence if G is a connected Lie group, then $B(G)$ satisfies the exact sequence

$$(1) \to K \to B(G) \to R^m \times D \to (1)$$

where K is the maximal compact connected normal subgroup of G and D is finitely generated abelian and central in G/K. In our case, G contains no compact normal semisimble subgroup. Thus $K = T^k$ for some integer k, where T is the unit circle in the complex plane. But then T^k is central; so we will consider $ZM(G)$ for G a connected Lie group with no compact connected normal semisimple subgroup. For a G, the subgroup $B(G)$ satisfies the central exact sequence

$$(1) \to T^k \to B(G) \to R^m \times D \to (1)$$

i.e. T^k is central, and $B(G)/T^k \cong R^m \times D$.

Greenleaf *et al.*[1] proved that all finite central measures on a connected Lie group G are supported on the the closed subgroup $B(G)$, *i.e.* $ZM(G) = Z^G M(B(G))$. So we need to study the algebra $Z^G M(B(G))$, of $I(G)$-invariant finite measures on $B(G)$.

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حول دراسة جبر القياسات المركزي $ZM(G)$ لزمرة لي الموصولة

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المستخلص . في هذا البحث تم تحويل زمرة لي الموصولة إلى زمرة لي الموصولة التي لا تحتوى على أي زمرة جزئية عمودية ملمومة شبه بسيطة .

وقد تم كذلك تقليص دراسة جبر القياسات المركزي $ZM(G)$ إلى دراسة ما هو أبسط منه ، وهو $Z^GM(B(G))$ جبر القياسات المحدودة العديمة التأثير *(G-invariant)* على الزمرة الجزئية المميزة $B(G)$ للزمرة G .

وفي النهاية أعطيت $B(G)$ لتحقق خواص المتتابعة التامة المركزية المذكورة في البحث