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Multivalued generalized nonlinear contractive maps and fixed points

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ABSTRACT

We introduce some notions of generalized nonlinear contractive maps and prove some fixed point results for such maps. Consequently, several known fixed point results are either improved or generalized including the corresponding recent fixed point results of Ciric [L.B. Ciric, Multivalued nonlinear contraction mappings, Nonlinear Anal. 71 (2009) 2716–2723], Klim and Wardowski [D. Klim, D. Wardowski, Fixed point theorems for set-valued contractions in complete metric spaces, J. Math. Anal. Appl. 334 (2007) 132–139], Feng and Liu [Y. Feng, S. Liu, Fixed point theorems for multivalued contractive mappings and multivalued Caristi type mappings, J. Math. Anal. Appl. 317 (2006) 103–112] and Mizoguchi and Takahashi [N. Mizoguchi, W. Takahashi, Fixed point theorems for multivalued mappings on complete metric spaces, J. Math. Anal. Appl. 141 (1989) 177–188].

1. Introduction and preliminaries

Let (X, d) be a metric space, 2^X a collection of nonempty subsets of X, and $CB(X)$ a collection of nonempty closed bounded subsets of X, $Cl(X)$ a collection of nonempty closed subsets of X, $K(X)$ a collection of nonempty compact subsets of X.

For any $A, B \in CB(X)$, let

$$H(A, B) = \max\{\sup_{x\in A} d(x, B), \sup_{y\in B} d(y, A)\},$$

where $d(x, B) = \inf_{y\in B} d(x, y)$. H is called the Hausdorff metric induced by d.

An element $x \in X$ is called a *fixed point* of a multivalued map $T : X \to 2^X$ if $x \in T(x)$. We denote $\mathrm{Fix}(T) = \{x \in X : x \in T(x)\}$. A sequence $\{x_n\}$ in X is called an *orbit* of T at $x_0 \in X$ if $x_n \in T(x_{n-1})$ for all $n \geq 1$.

A map $f : X \to \mathbb{R}$ is called *lower semicontinuous* if any sequence $\{x_n\} \subset X$ with $x_n \to x \in X$ implies that $f(x) \leq \liminf_{n\to\infty} f(x_n)$.

Investigations on the existence of fixed points of multivalued contractions in metric spaces were initiated by Nadler [1]. Using the Hausdorff metric, he established the following multivalued version of the well known Banach contraction principle.

Theorem 1.1. *Let (X, d) be a complete metric space and let $T : X \to CB(X)$ be a map such that for a fixed constant $h \in (0, 1)$ and for each $x, y \in X$,*

$$H(T(x), T(y)) \leq hd(x, y).$$

Then $\mathrm{Fix}(T) \neq \emptyset$.

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