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Some boundary value problems of fractional differential equations and inclusions

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ABSTRACT

In this paper, we study the existence of solutions for nonlinear fractional differential equations and inclusions of order $q \in (1, 2]$ with families of mixed and closed boundary conditions. In case of inclusion problems, the existence results are established for convex as well as nonconvex multivalued maps. Our results are based on Leray–Schauder degree theory, nonlinear alternative of Leray–Schauder type, and some fixed point theorems for multivalued maps. Some interesting special cases are also discussed.

1. Introduction

Differential equations and inclusions of fractional order have recently been addressed by several researchers for a variety of problems. The fractional calculus has found its applications in various disciplines of science and engineering such as physics, chemistry, biology, economics, control theory, signal and image processing, biophysics, blood flow phenomena, aerodynamics, fitting of experimental data, etc. [1–4]. For some recent work on fractional differential equations and inclusions, see [5–15] and the references therein.

In this paper, we consider for $T > 0$ and $1 < q \leq 2$ the following fractional differential equation

$$^cD^q x(t) = f(t, x(t)), \quad t \in [0, T], \tag{1.1}$$

where $^cD^q$ denotes the Caputo fractional derivative of order q and $f : [0, T] \times \mathbb{R} \to \mathbb{R}$. We study (1.1) subject to two families of boundary conditions:

(i) Mixed boundary conditions

$$Tx'(0) = -ax(0) - bx(T) \qquad Tx'(T) = bx(0) + \mathrm{d}x(T). \tag{1.2}$$

(ii) Closed boundary conditions

$$x(T) = \alpha x(0) + \beta Tx'(0), \qquad Tx'(T) = \gamma x(0) + \delta Tx'(0), \tag{1.3}$$

where $a, b, d, \alpha, \beta, \gamma, \delta \in \mathbb{R}$ are given constants.

Here we remark that the boundary conditions (1.2) interpolate between Neumann ($a = b = d = 0$) and Dirichlet ($a = b = d = \infty$) boundary conditions while (1.3) include quasi-periodic boundary conditions ($\beta = \gamma = 0$) and interpolate between periodic ($\alpha = \delta = 1, \beta = \gamma = 0$) and antiperiodic ($\alpha = \delta = -1, \beta = \gamma = 0$) boundary conditions. Notice that Zaremba boundary conditions $x(0) = 0, x'(T) = 0$ can be considered either as mixed boundary conditions with

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