Acta Mathematica Sinica, English Series
Jun., 2010, Vol. 26, No. 6, pp. 1165–1176
Published online: May 15, 2010
DOI: 10.1007/s10114-010-7521-0
Http://www.ActaMath.com

Acta Mathematica Sinica, English Series

Viscosity Approximation Methods for Nonexpansive Multimaps in Banach Spaces

Habtu ZEGEYE
Bahir Dar University, P. O. Box 859, *Bahir Dar, Ethiopia*
E-mail: *habtuzh@yahoo.com*

Naseer SHAHZAD
Department of Mathematics, King Abdul Aziz University, P. O. Box 80203, *Jeddah* 21589, *Saudi Arabia*
E-mail: *nshahzad@kau.edu.sa*

Abstract We prove strong convergence of the viscosity approximation process for nonexpansive nonself multimaps. Furthermore, an explicit iteration process which converges strongly to a fixed point of multimap T is constructed. It is worth mentioning that, unlike other authors, we do not impose the condition "$Tz = \{z\}$" on the map T.

Keywords nonexpansive retract, Banach spaces, fixed point, inwardness, nonexpansive multimap

MR(2000) Subject Classification 47H10, 47H09

1 Introduction

Let D be a nonempty closed convex subset of a Banach space $E := (E, \|\cdot\|)$ and let $K(D)$ and $KC(D)$ denote the family of nonempty compact subsets and nonempty compact convex subsets of D, respectively. Let $T : D \to K(E)$. Then T is said to be a *contraction* if there exists $0 \le k < 1$ such that $H(Tx, Ty) \le k\|x - y\|$ for $x, y \in D$, where H is the Hausdorff metric induced by $\|\cdot\|$. If $k = 1$, then T is called *nonexpansive*. A point x^* is a *fixed point of* T if $x^* \in Tx^*$. The set of fixed points of T is denoted by $F(T)$.

Let $T : D \to K(D)$ be nonexpansive. Given a $u \in D$ and a $t \in (0, 1)$, let $G_t : D \to K(D)$ be defined by

$$G_t x := tTx + (1 - t)u, \quad x \in D.$$

Then G_t is a contraction and, by the Nadler contraction principle [1], has a fixed point $x_t \in D$, that is,

$$x_t \in tTx_t + (1 - t)u. \tag{1.1}$$

Let

$$P_T(x) = \{u_x \in Tx : \|x - u_x\| = d(x, Tx)\},$$

where $d(x, A) := \inf\{\|x - a\| : a \in A \subset E\}$. Then $P_T : D \to K(E)$ is a multimap having nonempty compact values.

Received September 26, 2007, Accepted November 20, 2008